Received May 13, 1773.

XXXVI. *Properties of the Conic Sections; deduced by a compendious Method. Being a Work of the late* William Jones, *Esq; F. R. S. which he formerly communicated to Mr.* John Robertson, *Libr. R. S. who now addresses it to the Reverend* Nevil Maskelyne, *F. R. S. Astronomer Royal.*

S I R,

Read June 24, 1773.

YOU well know that the curves formed by the sections of a cone, and therefore called CONIC SECTIONS, have, from the earliest ages of geometry, engaged the attention of mathematicians, on account of their extensive utility in the solution of many problems, which were incapable of being constructed by any possible combination of right lines and circles, the magnitudes used in plane geometry. The properties of these curves are become far more interesting within the two last centuries, since they have been found to be similar to those which are described by the motions of the cœlestial bodies in the Solar system.

Two

Two different methods have been taken by the writers who have treated of their properties; the one, and the more antient, is to deduce them from the properties of the cone itself; the other is to consider the curves, as generated by the constant motion of two or more strait lines moving in a given plane, by certain laws.

There are various methods of generating these curve lines *in plano*; one method will give some properties very easily; but others, with much trouble: while, by another mode of description, some properties may be readily derived, which, by the former, were not so easily come at: so that it appears there may be a manner of describing the curves similar to the Conic Sections, by the motion of lines on a plane, which, in general, shall produce the most essential properties, with the greatest facility.

That excellent mathematician, the late William Jones, Esq; F. R. S. had drawn up some papers on the description of these curves, or lines of the second kind, very different from what he gave in his *Synopsis Palmariorum Matheseos*, published in the year 1706; or from that of any other writer on this subject. A copy of these papers he was pleased to let me take about the year 1740. He had not finished them as he intended; but, in their present state, they appear of too much consequence to be lost; as, it is much to be feared, his own copy, together with many other valuable papers, are; and therefore, I am desirous of preserving them in the Philosophical

 Trans-

Tranſactions, in the manner I at firſt tranſcribed them; although, I am aware, they might have been put into a form more pleaſing to the generality of readers: I have indeed annexed larger diagrams than what accompanied the author's copy, in order to render the lines more diſtinct, as all the relations are to be repreſented in a ſingle figure, of each kind.

Mr. Jones, having laid down a very ſimple method of deſcribing theſe curves, ſeems to have been deſirous of arriving at their properties in as expeditious a way as he could contrive; and therefore he has uſed the algebraic method, in general, of reducing his equations; and, on ſome occaſions, has uſed the method of fluxions, to deduce ſome properties chiefly relating to the tangents; and, by a judicious uſe of theſe, he has very much abridged the ſteps which otherwiſe he muſt have taken, to have deduced the very great variety of relations he has obtained: theſe he intended to have arranged in tables, from whence an equation expreſſing the relation between any three or more lines of the Conic Sections, might be taken out as readily as a logarithm out of their tables; this he has only partly executed; but it may eaſily be continued by thoſe who are deſirous to have it done, and are ſufficiently acquainted with what follows.

From the Houſe of the Royal Society,
April 29, 1773.

THE

THE DESCRIPTION OF LINES OF THE SECOND KIND.

LET the right lines AD, AQ, be drawn on a plane, at any inclination the one with the other. See PLATE XIV. Fig. 1, 2, 3.

In AD, AQ, take A*a*, AM, of any given magnitude, and draw MN parallel to AD,

On the points A, *a*, let two rulers AP, *a*P, revolve, and cut MN, AQ, in N and Q, fo that AQ be every-where equal to MN.

Then fhall the interfection P of the rulers defcribe lines of the fecond kind, or curves of the firft kind.

Where the right-line A*a*, is the firft, or tranfverfe diameter.

The point C, bifecting the diameter A*a*, is the center.

The right-line PD, drawn parallel to AQ, is the ordinate to the diameter A*a*.

The part AD, or CD, of the diameter, is the abfcifs, when reckoned to begin from A to C, or from C to A.

The right line B*b* drawn from the center C parallel to the ordinate PD, and terminated in the curve, is called the fecond, or conjugate diameter.

Thofe diameters to which the ordinates are perpendicular, are called the axes.

And AM is the parameter to the diameter A*a*.

THE

The properties of Lines of the second kind.

1. Put $Aa=d=2AC=2t$; $Bb=\delta=2BC=2c$; $AM=2p$; $PD=y$; $CD=x$; $AD=u$. Then $\overline{PD}^2=\frac{p}{t}\times ADa$.

Or $yy=\frac{p}{t}\times u\times\overline{d\mp u}=2pu\mp\frac{p}{t}uu=\frac{p}{t}\times\overline{\pm tt\mp xx}=\pm pt\mp\frac{p}{t}x^2$.

For $PD=\frac{AM\times AD}{MN}=\frac{AQ\times Da}{Aa}$ (by sim. $\triangle$s). Th. $\overline{PD}^2=\frac{AM}{Aa}\times ADa$.

2. Consequently $\frac{t}{p}yy=\pm tt\mp xx=du\mp uu=\pm\frac{1}{4}dd\mp xx$.

3. Hence $\frac{t}{p}y\dot{y}=\mp x\dot{x}=\overline{t\mp u}\times\dot{u}=x\dot{u}$.

4. And $pt=cc$, or $2pd=\delta\delta$; for when $y=c=\frac{1}{2}\delta$, then $x=0$.

5. Therefore $\frac{\overline{PD}^2}{ADa}=\frac{yy}{\pm tt\mp xx}=\frac{p}{t}=\frac{2p}{d}=\frac{2pd}{dd}=\frac{pt}{tt}=\frac{\delta\delta}{dd}=\frac{cc}{tt}=\frac{\overline{BC}^2}{\overline{AC}^2}$.

6. The curve line whose property is $yy+\frac{p}{t}uu-2pu=0$,
(where the abscissa begins at the curve),
Or $yy+\frac{cc}{tt}xx-cc=0$, (where it begins at the center),
is called an Ellipsis. This curve returns into itself. For when $x=0$, then $y=c$; and when $y=0$, then $x=t$. Which can happen but two ways.

7. The curve line whose property is $yy-\frac{p}{t}uu-2pu=0$,

Or $yy-\frac{cc}{tt}xx+cc=0$, is called an Hyperbola. This curve spreads out infinitely. For y increases as x increases; and has four legs tending contrary ways: for xx, or yy, may be produced as well from $-x$, or $-y$; as from $+x$, or $+y$.

8. If the point a, is supposed to be at an infinite distance from A, so that a ruler aP moves in a parallel position to AD; then is $yy=2pu$, or $yy-2pu=0$, the property of the curve described, and is called a Parabola. This curve spreads out infinitely; for y increases as u increases.

9. Let

9. Let Aa, Pp, be any two first diameters; Bb, Qq, their second diameters. PLATE XIV. Fig. 4.

Draw the ordinates PD, QE, to the diameter Aa, and the ordinate Pd, to the diameter B b.

Let PT be a tangent, and PM be perpendicular, to the curve, in P; PT cutting Aa, Bb, produced, in T, t; and PM, in M, m.

Put the subtangent DT $= s$, $dt = \sigma$.

Let AT $= r$, PM $= \pi$, CE $= w$, CT $= q = x \pm s = t \pm r$.

Put s, s', = sine and cosine of the angle MPD, or angle PmB.

R = tabular radius.

Then CDT $= xs = \overline{t \mp u} \times s = \frac{t}{p} yy = \frac{tt}{cc} yy = \pm tt \mp xx = u \times \overline{d \mp u} =$ ADa.

For $\dot{u} \times \overline{t \mp u} = \left(\frac{t}{p} y\dot{y} =\right) \alpha y\dot{y}$. Therefore $\left(\frac{\dot{u}}{\dot{y}} = \frac{\alpha y}{t \mp u} =\right) \frac{\alpha y}{x} = \frac{s}{y}$ (by sim. $\triangle$s).

And C$dt = y\sigma = \frac{p}{t} xx = \frac{cc}{tt} xx = \pm cc \mp yy =$ Bdb. For $\left(\frac{\dot{u}}{\dot{y}} =\right) \frac{\alpha y}{x} = \frac{x}{\sigma}$.

10. Hence $\overline{AC}^2 = tt = x \times \overline{x \pm s} = x \times \overline{t \pm r} = xq =$ DCT.

11. And $\overline{BC}^2 = cc = y \times \overline{y + \sigma} = y \times Ct = dCt$.

12. Consequently $\frac{\overline{Pd}^2}{Bdb} = \frac{xx}{\pm cc \mp yy} = \frac{tt}{cc} = \frac{\overline{AC}^2}{\overline{BC}^2}$.

13. Also $\overline{CE}^2 = ww = \left(\overline{DT}^2 \times \frac{\overline{QE}^2}{\overline{PD}^2} = \overline{DT}^2 \times \frac{AEa}{ADa} =\right) ss \times \frac{tt - ww}{sx} = sx = \frac{tt}{cc} yy = ADa = CDT$.

14. Therefore $AEa = \left(\frac{\overline{CE}^2}{\overline{DT}^2} \times ADa = \frac{sx}{ss} \times sx =\right) xx = \overline{CD}^2$.

15. And $\overline{QE}^2 = \left(\frac{\overline{PD}^2}{ADa} \times AEa =\right) \frac{cc}{tt} xx = \frac{p}{t} xx$.

In the general schemes. PLATE XV. and Fig. 5. PLATE XIV.

16. Let Aa, Bb, be the longest and shortest axes.
Draw Cϱ perpendicular to the tangent PT, cutting it in ϱ.
Put CP$=$T; CQ$=$C; C$\varrho=\varrho$.

Then $C\varrho=\varrho=\left(\frac{Ct\times CE}{CQ}=\frac{cc}{yy}\times\frac{ty}{cC}=\right)\frac{tc}{C}=\frac{AC\times CB}{CQ}$.

Hence the parallelogram, under the two axes, is equal to the parallelogram under any two diameters.

17. Draw the tangents AN, an, to any vertices A, a, meeting any diameters Pp, Qq, produced in V, U, and v, u, and the tangent PT in N, n.

Then $AU=\left(\frac{CA\times EQ}{CE}=\right)\frac{px}{p}$. And $AV=\left(\frac{CA\times PD}{CD}=\right)\frac{ty}{x}$.

18. Also $CU=\left(\frac{AU\times CQ}{EQ}=\right)\frac{cC}{y}$. And $CV=\left(\frac{CP\times CA}{CD}=\right)\frac{tT}{x}$.

19. Hence $PV=(CV \backsim CP=)\ T\times\frac{t\backsim x}{x}$. And $pV=T\times\frac{t+x}{x}$.

Also $QU=(CU\backsim CQ=)\ C\times\frac{c\backsim y}{y}$. And $qU=C\times\frac{c+y}{y}$.

20. When Aa and Bb are the longest and shortest axes; and when $y=p$,
Then $xx=tt\mp\frac{t}{p}yy$ will become $tt\mp pt=tt\mp cc$, which call ff.
And CD$=x$, will become CF$=$C$f=f$.
The points F, f, are called the Focii.

21. Hence $AF=af=\pm t\mp f$; $Af=aF=t+f$.

22. Also $\overline{CF}^2=\overline{Cf}^2=ff=\pm tt\mp cc=\pm tt\mp pt$.
And in the ellipsis, $\overline{AC}^2=\left(\overline{BC}^2+\overline{CF}^2=\right)\overline{BF}^2$.
in the hyperbola, $\overline{CF}^2=\left(\overline{AC}^2+\overline{BC}^2=\right)\overline{BA}^2$.
Hence, a circle described from B, with the distance AC in the ellipsis, or from C, with the distance AB in the hyperbola, will cut the axis Aa in the focii F, f.

23. Draw

23. Draw FP, fP, from the focii F, f, to any point P of the curve; and draw the conjugate diameters Pp, Qq.

Put PF $= z$; P$f = v$; PC$=$T; QC$=$C; $\frac{f}{t}=\varphi$; $\frac{f}{c}=\gamma$. Then

$$\overline{Pf}^2 = vv = yy + xx + 2xf + ff = tt + 2fx + \frac{ff}{tt}xx = TT + ff + 2fx.$$

$$\overline{PF}^2 = zz = yy + xx - 2xf + ff = tt - 2fx + \frac{ff}{tt}xx = TT + ff - 2fx.$$

$$\text{For } yy = \left(\frac{cc}{tt}\times\overline{tt \backsim xx} = \right)\frac{tt \backsim ff}{tt}\times\overline{tt \backsim xx}.$$

24. $$Pf = v = \left(\sqrt{tt + 2fx + \frac{ff}{tt}xx} = \right) t + \frac{f}{t}x = t + \varphi x = \frac{tt + fx}{t}.$$

$$PF = z = \left(\sqrt{tt - 2fx + \frac{ff}{tt}xx} = \right) t - \frac{f}{t}x = t - \varphi x = \frac{tt - fx}{t}.$$

25. $$PF \pm Pf = z \pm v = 2t = Aa.$$

26. $$\overline{Pf}^2 + \overline{PF}^2 = vv + zz = 2yy + 2xx + 2ff = 2tt + 2\varphi\varphi xx = 2TT + 2ff.$$

27. $$\overline{Pf}^2 - \overline{PF}^2 = vv - zz = 4fx = \overline{v+z}\times\overline{v-z} = 2t\times\overline{v\mp z}.$$

28. $$FPf = zv = z\times\overline{2t\mp z} = 2tz \mp zz = tt \backsim \varphi\varphi xx = \overline{tt \backsim xx}\times\frac{cc}{tt}xx$$

$$= cc \backsim \frac{ff}{cc}yy = \frac{tt}{cc}yy + \frac{cc}{tt}xx = \overline{CE}^2 + \overline{EQ}^2$$

$$= \overline{CQ}^2 = CC = T\times P.$$

29. Let $m = t - z = v - t = \frac{fx}{t} = \varphi x = \pm\frac{1}{2}v \mp \frac{1}{2}z$

$$= \frac{fx}{\sqrt{cc+ff}} = \frac{x}{t}\sqrt{tt\mp cc} = \sqrt{tt\mp zv}$$

$$= \sqrt{tt \mp CC} = \sqrt{ff + cc - zv}.$$

30. Hence $\dot{z} = -\dot{v} = \varphi\dot{x} = -\frac{f}{t}\dot{x} \pm \frac{fy}{px}\dot{y}$.

And $\dot{v} = -\dot{z} = \varphi\dot{x} = \frac{f}{t}\dot{x} = -\frac{fy}{px}\dot{y}$.

31. $$AC = t = \tfrac{1}{2}z \pm \tfrac{1}{2}v = \frac{cc}{p} = \sqrt{cc \pm ff} = \frac{fx}{m} = \frac{cx}{\sqrt{cc \backsim yy}}$$
$$= \frac{c}{y}\sqrt{sx} = \sqrt{x \times \overline{x \backsim s}} = \frac{y^2 + \sqrt{y^4 + 4p^2x^2}}{2p} = \frac{yy}{2p} + \sqrt{\frac{y^4}{4p^2} + xx}.$$

32. $$\overline{CD}^2 = \overline{Pd}^2 = xx = \overline{\frac{ttyy}{ccs}}\Big|^2 = \overline{\frac{tyy}{ps}}\Big|^2 = \pm tt \mp \frac{t}{p}yy = \frac{tt}{cc} \times \overline{cc \mp yy}$$
$$= \frac{ff+cc}{cc} \times \overline{cc \mp yy} = tt \mp \frac{tt}{cc}yy = \frac{mm}{\varphi\varphi} = \frac{tt}{ff}mm$$
$$= \frac{cc+ff}{ff}mm = \frac{tt}{ff} \times \overline{tt \mp zv} = \frac{tt}{ff} \times \overline{tt \mp CC} = \overline{\frac{tt}{q}}\Big|^2$$
$$= \frac{tt}{cc} \times zv - \frac{tt}{cc}yy = \frac{t}{p} \times zv - \frac{t}{p}yy = \frac{tt}{ff} \times \overline{t - z}\big|^2$$
$$= \frac{tt}{ff} \times \overline{v - t}\big|^2 = \overline{\frac{tt \mp tz}{f}}\Big|^2 = \overline{t \mp u}\big|^2 = \overline{\sqrt{tt + \tfrac{1}{4}ss} - \tfrac{1}{2}s}\Big|^2$$
$$= \frac{tt}{ff} \times tt \mp \frac{tt}{cc}\pi\pi = \frac{t^4}{c^4} \times \frac{SS}{RR}\pi\pi.$$

33. And $$\dot{x} = -\frac{ty\dot{y}}{px} = -\frac{tty\dot{y}}{ccx} = -\frac{tt\dot{y}}{qq} = -\frac{t\dot{z}}{f} = \frac{t\dot{v}}{f}.$$

34. $$\overline{PD}^2 = \overline{Cd}^2 = yy = \frac{p}{t}sx = \frac{cc}{tt}sx = \frac{cc}{tt} \times \overline{\pm tt \mp xx} = \pm cc \mp \frac{cc}{tt}xx$$
$$= \pm pt \mp \frac{p}{t}xx = \frac{2cc}{t}u \mp \frac{cc}{tt}uu = \frac{\pm tt \mp ff}{tt} \times \overline{\pm tt \mp xx}$$
$$= \frac{cc}{tt} \times zv - \frac{cc}{tt}xx = \frac{cc}{tt} \times CC - \frac{cc}{tt}xx = \frac{cc}{ff} \times \overline{zv - cc}$$
$$= \frac{cc}{ff}nn = \frac{pt}{ff}nn \text{ (putting } nn = zv - cc\text{)}$$
$$= \frac{tt\pi\pi - c^4}{ff} = cc \mp \frac{tt}{cc} \times \frac{SS}{RR}\pi\pi = \frac{S'S'}{RR}\pi\pi.$$

35. And $$\dot{y} = -\frac{px\dot{x}}{ty} = -\frac{ccx\dot{x}}{tty} = \frac{px\dot{z}}{fy} = -\frac{px\dot{v}}{fy}.$$

36. Also $$\frac{ff}{cc}yy = zv - cc = \frac{ff}{tt} \times \overline{tt \mp xx} = ff \mp \frac{ff}{tt}xx$$
$$= nn = ff \mp mm = CC - cc.$$

37. $\overline{PC}^2$

3

37. $\overline{PC}^2 = TT = xx + yy = xx \pm cc \mp \frac{cc}{tt}xx = tt - ff + \frac{ff}{tt}xx = cc + \varphi\varphi xx$
$= cc + tt \backsim zv = cc + mm = cc + tt \backsim CC$
$= tp + \varphi\varphi xx = 2cc + ff \backsim zv = 2pt + ff \backsim zv.$

38. And $\dot{T} = \frac{\varphi\varphi x\dot{x}}{T} = \frac{\varphi\varphi x\dot{x}}{\sqrt{tp+\varphi\varphi xx}} = \left(\frac{\varphi\varphi x}{T} \times -\frac{ty\dot{y}}{px} = \right)\frac{ffy\dot{y}}{ptT} = -\frac{ffy\dot{y}}{ccT}.$

39 Alſo $\overline{CQ}^2 + \overline{CP}^2 = (cc+tt=)\ \overline{CB}^2 + \overline{CA}^2.$

40. $CT = q = \frac{tt}{x} = \left(tt \times \frac{f}{tm} = \right)\frac{ft}{m} = \frac{ft}{t \backsim z} = \frac{ft}{\sqrt{tt \backsim zv}}$
$Ct = \frac{cc}{y} = \frac{cf}{n} = \frac{cf}{\sqrt{zv - cc}}.$

41. And $\dot{q} = -\frac{q\dot{x}}{x} = -\frac{qq\dot{x}}{tt} = \frac{qqy\dot{y}}{ccx} = \frac{qqy\dot{y}}{ptx} = -\frac{tt\dot{x}}{xx}$ (for $tt = qx$).

42. $AT = (\pm CT \mp CA =) \pm \frac{tt}{x} \mp t = \frac{t}{x} \times \pm \overline{t \mp x} = \pm \frac{ft}{m} \mp t = \pm \frac{ft}{t \mp z} \mp t.$
$aT = (CT + Ca =) \frac{tt}{x} + t = \frac{t}{x} \times \overline{t + x} = \frac{ft}{m} + t = \frac{ft}{t \backsim z} + t.$

43. $FT = \left(\pm CT \mp CF = \pm \frac{ft}{m} \mp f = \frac{\pm ft \mp fm}{m}\right) = \frac{fz}{t \backsim z} = \frac{fz}{m} = \frac{tz}{x}.$
$fT = \left(CT + Cf = \frac{ft}{m} + f = \frac{ft + fm}{m} = \right) \frac{fv}{v \backsim t} = \frac{fv}{m} = \frac{tv}{x}.$

On A*a* deſcribe a circle, draw the tangent TP', and draw CP'.
Continue DP to P', and at right angles to TP', draw FR',

Then $FR' = \left(\frac{FT}{CT} \times CP' = \frac{tz}{x} \times t \times \frac{x}{tt} = \right) z = FP.$
$fr' = \left(\frac{fT}{CT} \times CP' = \frac{tv}{x} \times t \times \frac{x}{tt} = \right) v = fP.$

44. $DT = s = \frac{tyy}{px} = \frac{ttyy}{ccx} = \frac{\pm tt \mp xx}{x} = \frac{tnn}{fm}$.

$$dt = \sigma = \frac{pxx}{ty} = \frac{ccxx}{tty} = \frac{cc \mp yy}{y} = \frac{cmm}{fn}.$$

45. $AN = \left(\frac{Ct}{CT} \times AT =\right) \frac{cm}{nx} \times \overline{t - x} = \frac{c}{n} \times \overline{f - m}$.

$$an = \left(\frac{Ct}{CT} \times aT =\right) \frac{cm}{nx} \times \overline{t + x}.$$

46. $VN = \left(\frac{PV \times Ct}{PC} = \frac{t - x}{T} \times \frac{T}{x} \times \frac{fc}{n} =\right) \frac{fc}{nx} \times \overline{t - x} = \frac{fc}{nm} \times \overline{f - m}$

$$= \frac{fc}{\sqrt{zv - cc} \times \sqrt{tt - zv}} \times \overline{f - \sqrt{tt - zv}}$$

$$UN = Ct = \frac{cc}{y}.$$

47. $AD = (\pm AC \mp CD = \pm t \mp \frac{tm}{f} =) \frac{t}{f} \times \overline{\pm f \mp m}$

$$aD = (AC + CD = t + \frac{tm}{f} =) \frac{t}{f} \times \overline{f + m}.$$

48. Produce PF, pf, to meet the curve in Π, π;

Draw $\Pi\Delta$ perpendicular to Aa.

Put $F\Pi = z'$, $f\pi = v'$, $F\Delta = x'$.

Now, $FD = (\pm CF \mp CD =) \pm f \mp x = \pm f \mp \frac{tm}{f} = \frac{tz - cc}{f}$

$$fD = f + \frac{tm}{f} = \frac{tv - cc}{f}.$$

Then $F\Delta = x' = (C\Delta - CF =) \frac{tm}{f} - f = \left(\frac{t}{f} \times \overline{t - z'} - f =\right) \frac{cc - tz}{f}$

$$f\Delta = f + \frac{tm}{f} = \frac{tv' - cc}{f}.$$

49. From

49. From F, f, draw FR, fr, perpendicular to the tangent PT, aud cutting it in R, r.

$$FR = \lambda = \frac{FT \times C\varrho}{CT} = \frac{cz}{C} = \frac{cz}{\sqrt{zv}} = \frac{cz}{\sqrt{2tz - zz}} = c\sqrt{\frac{z}{v}}$$
$$= c\sqrt{\frac{t - \varphi x}{t + \varphi x}} = \sqrt{\frac{ptz}{2t - z}} = \frac{z\varrho}{t}$$
$$fr = \lambda' = \frac{fT \times C\varrho}{CT} = \frac{cv}{C} = \frac{cv}{\sqrt{zv}} \cdots\cdots = c\sqrt{\frac{z}{v}}$$
$$= e\sqrt{\frac{t + \varphi x}{t - \varphi x}} = \sqrt{\frac{ptv}{2t - v}}.$$

$\dot{\lambda} = \frac{tc\dot{z}}{v\sqrt{zv}}$. For $\lambda^2 = \frac{c^2 z}{v}$. Th. $2\lambda\dot{\lambda} = \left(\frac{c^2 v\dot{z} - c^2 z\dot{v}}{vv} =\right) \frac{2c^2 t\dot{z}}{vv}$.

50. $TR = \left(\frac{AT \times RF}{AN} = \frac{tnz}{mC} =\right) \frac{ttnz}{fcx} = \frac{ttn}{fx}\sqrt{\frac{z}{v}} = \frac{tn}{m}\sqrt{\frac{z}{v}}$.

$Tr = \frac{aT \times rf}{an} = \frac{tn}{m}\sqrt{\frac{v}{z}}$.

51. Draw PM perpendicular to the tangent PT, meeting the axes Aa, Bb, in M, m.

$$DM = \left(\frac{PD \times PD}{TD} =\right) \frac{ccm}{ft} = \frac{cc}{ft}\sqrt{tt \pm zv} = \frac{cc}{tt}x = \frac{p}{t}x$$
$$= \frac{S}{R}\pi = p \mp \frac{p}{t}u = \frac{cc}{tf}\sqrt{tt \mp \frac{tt}{cc}\pi\pi}.$$
$$dm = \left(\frac{PD \times Pd}{DM} =\right) \frac{ttn}{fc} = \frac{tt}{cc}\sqrt{zv - cc} = \frac{tt}{cc}y = \frac{t}{p}y.$$

52. $CM = \left(CD \mp DM = x \mp \frac{cc}{tt}x =\right) \frac{tt \mp cc}{tt}x = \frac{ff}{tt}x = \frac{f}{t}m$

$$= \frac{f}{t}\sqrt{tt \mp zv} = \frac{f}{t} \times \overline{t \mp z} = \frac{f}{t} \times \sqrt{tt \mp \frac{tt}{cc}\pi\pi}$$

$cm = \left(dm \mp Cd = \frac{tt}{cc}y \mp y =\right) \frac{tt \mp cc}{cc}y = \frac{ff}{cc}y = \frac{f}{c}n = \frac{f}{c}\sqrt{zv - cc}$

53. $FM = \left(CF \backsim CM = f \backsim \frac{fm}{t} =\right) \frac{f}{t} \times \overline{t \mp m} = \frac{fz}{t} = \pm f \mp \frac{ff}{tt}x.$

$fM = \left(fC + CM = f + \frac{fm}{t} =\right) \frac{f}{t} \times \overline{t + m} = \frac{fv}{t} = f \pm \frac{ff}{tt}x.$

54. $\overline{Fm}^2 = \overline{fm}^2 = (\overline{cf}^2 + \overline{cm}^2 =)\ ff + \frac{ffnn}{cv} = \frac{ffzv}{cc} = \frac{ff}{cc} \times \text{cc}.$

55. $\text{AM} = (\text{AC} \backsim \text{CM} =)\ t \backsim \frac{fm}{t} = \frac{tt \backsim fm}{t} = t \backsim \frac{ff}{tt}x.$

$a\text{M} = (a\text{C} + \text{CM} =)\ t + \frac{fm}{t} = \frac{tt + fm}{t} = t + \frac{ff}{tt}x.$

56. $\text{TM} = (\text{TF} + \text{FM} = \frac{fz}{m} + \frac{fz}{t} =)\ \frac{fzv}{tm} = \frac{zv}{x} = \frac{\text{cc}}{x}$

$tm = (\text{C}t + \text{C}m = \frac{fc}{n} + \frac{fn}{c} =)\ \frac{fzv}{cn} = \frac{zv}{y} = \frac{\text{cc}}{y}.$

57. $\text{PM} = \pi = \left(\frac{\text{FR} \times \text{TM}}{\text{FT}} =\right) \frac{c}{t}\sqrt{zv} = \frac{c}{t}\text{c} = \frac{p}{c}\text{c} = \frac{c}{t}\sqrt{tt \backsim \varphi^2 x^2}$

$= \frac{c}{t}\sqrt{\overline{t + \varphi x} \times \overline{t - \varphi x}} = \frac{cc}{tt} \times \frac{\text{R}}{\text{S}}x = \frac{c\text{R}}{ts}\sqrt{ce \mp yy}$

$= \frac{c}{tt}\sqrt{t^4 - ffxx} = \frac{c}{t}\sqrt{cc + \gamma\gamma yy} = \frac{1}{t}\sqrt{c^4 + ffyy}$

$= \frac{c}{t}\sqrt{2tz \mp zz} = \sqrt{2pz \mp \frac{p}{t}zz}.$

$pm = \pi' = \left(\frac{\text{PM} \times \text{P}d}{\text{DM}} =\right) \frac{t}{c}\sqrt{zv} = \frac{t}{c}\text{c} = \frac{c}{p}\text{c} = \frac{t}{c}\sqrt{tt \backsim \varphi\varphi xx}$

$= \frac{t}{c}\sqrt{\overline{t + \varphi x} \times \overline{t - qx}} = \frac{1}{c}\sqrt{t^4 \backsim ffxx} = \frac{t}{c}\sqrt{cc + \gamma\gamma yy}$

$= \frac{t}{cc}\sqrt{c^4 + ffyy} = \frac{t}{c}\sqrt{2tz \mp zz} = \sqrt{\frac{2tt}{p}z \mp \frac{t}{p}zz}.$

58. $\text{M}m = (\text{P}m \mp \text{PM} = \frac{t}{c}\text{c} \mp \frac{c}{t}\text{c} =)\ \frac{tt \mp cc}{ct}\text{c} = \frac{ff}{ct}\sqrt{zv}.$

$= \frac{ff}{ct}\sqrt{tt \backsim \varphi\varphi xx}.$

59. $\text{PT} = \tau = \left(\frac{\text{PD} \times \text{PM}}{\text{DM}} =\right) \frac{n}{m}\text{c} = \sqrt{\frac{zv - cc}{tt - zv}} \times zv = \frac{tn}{fx}\text{c}.$

$= \frac{ty}{cx}\text{c} = \frac{ty}{cx}\sqrt{tt \backsim \varphi\varphi xx} = \frac{\sqrt{\overline{zv - cc} \times zv}}{t \mp x}.$

$\text{P}t = \tau' = \left(\frac{\text{PT} \times \text{P}d}{\text{DT}} =\right) \frac{m}{n}\text{c} = \sqrt{\frac{tt - zv}{zv - cc}} \times vz = \frac{fx}{tn}\text{c}$

$= \frac{cx}{ty}\text{c} = \frac{cx}{ty}\sqrt{tt - \varphi\varphi xx}.$

60. TN

60. $\text{TN} = \left(\frac{\text{PT}\times\text{TA}}{\text{DT}} = \frac{n\text{C}}{m}\times\frac{t}{m}\times\overline{f \backsim m}\times\frac{fm}{tnn} = \right)\frac{f\text{C}}{mn}\times\overline{f\backsim m} = \frac{c\text{C}}{xy}\times\overline{t\backsim x}$

$\text{T}n = \left(\frac{\text{PT}\times\text{T}a}{\text{DT}} = \frac{n\text{C}}{m}\times\frac{t}{m}\times\overline{f+m}\times\frac{fm}{tnn} = \right)\frac{f\text{C}}{mn}\times\overline{f+m} = \frac{c\text{C}}{xy}\times\overline{t+x}$

61. $\text{PN} = \left(\frac{\text{AD}\times\text{PT}}{\text{DT}} = \frac{t}{f}\times\overline{f\backsim m}\times\frac{n\text{C}}{m}\times\frac{fm}{tnn} = \right)\frac{\text{C}}{n}\times\overline{f\backsim m} = \frac{c\text{C}}{ty}\times\overline{t\backsim x}$

$\text{P}n = \left(\frac{a\text{D}\times\text{PT}}{\text{DT}} = \frac{t}{f}\times\overline{f+m}\times\frac{n\text{C}}{m}\times\frac{fm}{tnn} = \right)\frac{\text{C}}{n}\times\overline{f+m} = \frac{c\text{C}}{ty}\times\overline{t+x}$

62. $\text{PR} = \left(\frac{\text{PT}\times\text{FM}}{\text{TM}} = \right)\frac{nz}{\text{C}} = n\sqrt{\frac{z}{v}} = \sqrt{\frac{z}{v}\times\overline{zv-cc}} = \frac{f}{c}y\times\sqrt{\frac{z}{v}}$

$$= \frac{f}{c}y\times\sqrt{\frac{t-\varphi x}{t+\varphi x}} = \frac{f}{cc}\times\lambda y$$

$\text{P}r = \left(\frac{\text{PT}\times f\text{M}a}{\text{TM}} = \right)\frac{nv}{\text{C}} = n\sqrt{\frac{v}{z}} = \sqrt{\frac{v}{z}\times\overline{zv-cc}} = \frac{f}{c}\times y\sqrt{\frac{v}{z}} = \frac{f}{c}y\times\sqrt{\frac{t+\varphi x}{t-\varphi x}}$

63. $\text{T}\varrho = \left(\frac{\text{PT}\times\text{TC}}{\text{TM}} = \right)\frac{ntt}{m\text{C}} = \frac{tt}{t-z}\sqrt{\frac{zv-cc}{vz}} = \frac{fytt}{mc\text{C}} = \frac{yt^3}{xc\text{C}}$

64. $\text{P}\varrho = \left(\text{T}\varrho \mp \text{TP} = \frac{ntt}{m\text{C}} \mp \frac{n\text{C}}{m}\times\right)\frac{n}{m\text{C}}\times tt \mp \text{CC} = \frac{n}{m\text{C}}\times mm = \frac{nm}{\text{C}}.$

65. $\text{T}t = \left(\frac{\text{PT}\times\text{C}t}{\text{PD}} = \frac{n\text{C}}{m}\times\frac{cf}{n}\times\frac{f}{cn} = \right)\frac{ff\text{C}}{mn} = ff\sqrt{\frac{zv}{\overline{tt-zv}\times\overline{zv-cc}}}$

$$= \frac{tc}{xy}\text{C} = \frac{tc}{xy}\sqrt{tt-\frac{ff}{tt}xx}$$

66. $\text{N}n = \left(\frac{\text{PT}\times\text{A}a}{\text{DT}} = \right)\frac{2f\text{C}}{cn} = \frac{2c}{y}\text{C} = \frac{2c}{y}\sqrt{zv} = 2f\sqrt{\frac{zv}{zv-cc}}$

From C and F, draw Cγ and Fϵ, perpendicular to Pf
Put Pϵ $= b$; $\Sigma =$ *co*-fine of the $\angle$ AfP; R $=$ tabular radius..

Then $f\gamma = \left(\frac{\text{C}f\times f\text{D}}{\text{P}f} = \frac{tv-cc}{f} = \right)\frac{tv-cc}{v} = \left(\frac{\Sigma}{\text{R}}\times\text{C}f\right)\frac{\Sigma f}{\text{R}}$

And $\text{P}f = v = \frac{cc}{t-\frac{\Sigma}{\text{R}}f} = \frac{tt-ff}{t-\frac{\Sigma}{\text{R}f}}$

$$\text{PS} = h = \left(\frac{\pm zz + vv \mp 4ff}{2v} = \frac{\pm \text{TT} \mp ff}{v} = \frac{\pm 2tt \mp vz}{v} = \right)$$
$$= \pm \frac{2pt}{v} \mp z = \pm \frac{z+v}{v} p \mp z.$$

Hence $p = \frac{\pm h \mp z}{z+v} \times v$; $v = \frac{pz}{\mp h \pm z - p}$; $z = \frac{p-h}{v-p} \times v$

Therefore p is less than, equal to, or greater than $\pm h \mp z$, in the Ellipsis, Parabola, or Hyperbola.

67. Draw $m\mu$, making the $\angle \text{P}m\mu = \angle \text{PCM}$, and cutting PC in μ.

Then $\text{P}\mu = \left(\frac{\text{PM} \times \text{P}m}{\text{PC}} = \right) \frac{\text{CC}}{\text{T}} = \frac{zv}{\sqrt{tt \pm cc \mp zv}} = \frac{tt - \varphi\varphi xx}{\sqrt{cc + \varphi\varphi xx}}$
$= \text{P} = \frac{1}{2}$ parameter to PC.

68. Let FΦ be an ordinate to the axis Aa. at the focus F. and ΦG a tangent to the curve in Φ, meeting Aa, BC, AN, an, in G, g, S, s. Then

$$\text{F}\Phi = \sqrt{\frac{cc}{tt} \times \text{AF}a} = \sqrt{\frac{cc}{tt} \times cc} = \Big) \frac{cc}{t} = \frac{pt}{t} = p$$
$= \frac{1}{2}$ parameter to Aa.

69. $\text{CG} = \frac{tt}{f} = \frac{ff \pm cc}{f} = \frac{tt}{\sqrt{tt - cc}} = \frac{tt}{\sqrt{t \times t - p}}$

70. $\text{FG} = (\text{CG} \backsim \text{CF} = \frac{tt}{f} \backsim f = \frac{tt \backsim ff}{f} = \frac{cc}{f} = \frac{pt}{f}.$

71. Draw PH parallel to Aa; and GH perpendicular to Aa, meeting, PH, PT, in H, h; then

$$\text{PH} = \text{DG} = (\text{CG} \backsim \text{DC} =) \frac{tt}{f} \backsim x = \frac{t}{f} z = \frac{t}{f} \times \text{PF}.$$

72. TO

72. $\text{TO} = (\text{CT} \backsim \text{CG} = \frac{ft}{m} \backsim \frac{tt}{f} = \frac{t}{m} \times \overline{f \backsim x} =) \frac{t}{t-z} \times \frac{\pm tz \mp cc}{f} = \frac{tt}{fx} \times \overline{f \backsim x}.$

73. $\text{G}h = (\frac{\text{PD} \times \text{TO}}{\text{DT}} = \frac{cn}{f} \times \frac{t}{m} \times \overline{f \backsim x} \times \frac{fm}{tnn} = \frac{\text{C}}{n} \times \overline{f \backsim x} =)$

$$\frac{cc}{fy} \times \overline{f \backsim x} = \frac{c}{\sqrt{zv - cc}} \times \frac{tz - cc}{f}.$$

74. $\text{P}h = (\frac{\text{TP} \times \text{DG}}{\text{DT}} =) \frac{z}{n}\text{C} = \frac{cz}{fy}\text{C} = z \times \sqrt{\frac{zv}{zv - cc}} = \frac{cz}{fy} \sqrt{zv}.$

75. $\text{R}h = (\text{P}h - \text{PR} =) \frac{z\text{C}}{n} - \frac{zn}{\text{C}} = \frac{z}{n\text{C}} \times \overline{\text{CC} - nn} =)$

$$\frac{zcc}{n\text{C}} = \frac{zcc}{\sqrt{zv - cc + zv}} = \frac{zc^3}{fy\text{C}}$$

76. $\text{F}h = (\sqrt{\overline{\text{FR}}^2 + \overline{\text{R}h}^2} =) \frac{cz}{n} = \frac{cz}{\sqrt{zv - cc}} = \frac{ccz}{fy} = \frac{ptz.}{fy}$

77. Let $\text{D}\Sigma$ be any Ordinate to the axe $\text{A}a$, cutting the curve in Σ, and the focal tangent ΦG in σ;

Then $\text{D}\sigma = (\frac{\text{F}\Phi + \text{DG}}{\text{FG}} = \frac{cc}{t} \times \frac{tz}{f} \times \frac{f}{cc} =) z = \text{FP} = \text{F}\Sigma.$

78. Therefore $\text{AS} = \text{AF}$; $as = a\text{F}$; $\text{C}g = \text{CA}$; by sim. $\triangle$s.

79. $\overline{\text{DF}}^2 = (\overline{\text{D}\sigma}^2 - \overline{\text{D}\Sigma}^2 = \overline{\text{D}\sigma + \text{D}\Sigma} \times \overline{\text{D}\sigma - \text{D}\Sigma} =) \text{P}\sigma \times \sigma\Sigma.$

79. Let the tangents PN, $p\text{L}$, to the opposite vertices P, p, cut the tangents AN, an, to the opposite vertices A, a, in N, n, L, l.
Then $\text{P}n = p\text{L}$; $an = \text{AL}$; $\text{PN} = pl$; $\text{AN}\, al$.
For the Trapezia's $\text{PC}an$, $p\text{CAL}$, are similar and equal;
And so are the Trapezia's PCAN, $p\text{C}al$.

81. $PF \times Mf = \left(\frac{f}{t} \times vz =\right) Pf \times MF = \frac{f}{t} \times FPf.$

82. $CEQ = (xy =)\ CDP = dCD = DM \times dm.$

83. $AN \times aD = \frac{ctn}{f} = an \times AD = AC \times PD = Dal = DAL.$

84. $RPr = nn = TP\varrho.$

85. $RTr = \left(\frac{nn}{mm} \times tt = \frac{tt - xx}{xx} \times tt =\right) ATa.$

86. $RTf = \left(\frac{tnz}{mc} \times \frac{fv}{m} = \frac{ft}{m} \times \frac{nc}{m} =\right) CTP = rTF.$

87. $NTn = \left(\frac{ffcc}{mm\,nn} \times \overline{ff \backsim mm} =\right) \frac{ffcc}{mm} = PTf.$

88. $PN \times CB = \left(\overline{t \backsim x} = \frac{fc}{tn} c =\right) AN \times CQ.$

89. $CAV = \left(t \times \frac{ty}{x} = \frac{tt}{x} \times y =\right) CT \times PD.$

90. $PRh = \left(\frac{cczz}{cc} =\right) \overline{FR}^2.$

Hence Fh is perpendicular to FP.

91. $EMf = \left(\frac{fz}{t} \times \frac{fv}{t} =\right) \frac{ff}{tt} \times zv = \frac{ff}{tt} \times cc = \frac{ff}{tt} \times \overline{CQ}^2$

92. $AMa = \left(tt \backsim \frac{ffmm}{tt} =\right) cc \times \frac{ff}{tt} \times zv$

$= \overline{CB}^2 + \frac{ff}{tt} \times \overline{CQ}^2 = \overline{CB}^2 + EMf.$

93. $\frac{\overline{PN}^2}{\overline{TN}^2}=\frac{xx}{tt}=\frac{\overline{CD}^2}{\overline{AC}^2}=\frac{\overline{AN}^2}{\overline{VN}^2}=\frac{\overline{CA}^2}{\overline{CT}^2}=\frac{ADa}{ATa}=\frac{ADa}{RTr}=\frac{\overline{AD}^2}{\overline{AT}^2}=\frac{\overline{aD}^2}{\overline{aT}^2}=\frac{CDT}{ATa}.$

94. $\frac{FT}{CT}=\frac{z}{t}=\frac{FP}{CA}=\frac{FR}{C\varrho}.$ Sim. $\triangle$s TFR, TCϱ.

And $\frac{AD}{DT}=\frac{x}{t+x}=\frac{CD}{aD}=\frac{AC}{aT}.$

95. $\frac{fT}{CT}=\frac{v}{t}=\frac{fP}{AC}=\frac{fr}{C\varrho}.$ And $\frac{AT}{DT}=\frac{t}{t+x}=\frac{AC}{aD}=\frac{CT}{aT}.$

96. $\frac{PM}{Pm}=\frac{cc}{tt}=\frac{p}{t}=\frac{\overline{BC}^2}{\overline{AC}^2}.$ And $\frac{Pm}{fm}=\frac{t}{f}=\frac{AC}{CF}.$

97. $\frac{FMf}{\overline{PM}^2}=\frac{ff}{cc}=\frac{\overline{t+c}\times\overline{t-c}}{cc}=\left(\frac{tt-tp}{tp}=\right)\frac{t-p}{p}=\frac{\overline{CF}^2}{\overline{CB}^2}.$

98. Let PM, the perpendicular to the tangent PT, cut the axis Aa in M; and fP, produced, cut FR in φ.
Then will PM bisect the angle FPf.
For PF $\times$ Mf=P$f\times$ MF.

99. And the angle FPT is equal to the angle fPt.
For $\angle$TPF + $\angle$FPM=$\angle t$Pf+ $\angle f$PM.

100. Therefore PT will bisect the angle FPφ.
For $\angle\varphi$PT= ($\angle f$Pt=) $\angle$FPT.

101. Consequently Pφ=PF. And Rφ=RF.

102. A circumference of a circle described from t, with the radius tN, will cut the axis Aa in the focii F, f.
For TN $\times$ Tn = TF $\times$ Tf.

 103. A

103. A circumference of a circle described from C, with the radius CA, will cut the tangent PT in R, r.

Whence the perpendiculars RF, rf, to that tangent, will cut the axis Aa in the focii F, f.

For $TR \times Tr = TA \times Ta$.

104. A circumference described from B, with AC, in the Ellipsis, or from C, with AB in the Hyperbola, will cut the axis Aa, in the focii F, f.

For, in the Ellipsis, $tt = cc + ff$, or $\overline{AC}^2 = (\overline{BC}^2 + \overline{CF}^2 =) \overline{BF}^2$.

And in the Hyperbola, $ff = tt + cc$, or $\overline{CF}^2 = (\overline{AC}^2 + \overline{BC}^2 =) \overline{AB}^2$.

105. Let CQ produced, cut PF, Pf, in z, x; draw mz, mx, and MZ, MX, parallel to them, cutting PF, Pf in Z, X.

Then $Px = CR = CA = Pz = t$.

106. Hence $\angle Pxz = \angle Pzx$; and $\angle mxz = \angle mzx$.

For Pm is perpendicular to zx.

Consequently, the angles Pzm, Pxm; PZM, PXM are equal.

107. And the triangles Pzm, Pxm, are similar and equal:

And so are the triangles PZM, PXM.

Consequently, the trapezias Pzmx, PZMX, are similar.

108. Let CR, Cr, cut PF, Pf, in K, k.

Then $CK = \left(\frac{CF \times Pf}{Ff} =\right) \frac{1}{2} Pf = Pk = kr$.

And $Ck = \left(\frac{Cf \times PF}{Ff} =\right) \frac{1}{2} PF = PK = KR$.

109. Also $PX = PZ = \left(\frac{PM}{Pm} \times Pz = \frac{cc}{t} \times \frac{c}{tC} \times t =\right) \frac{cc}{t} = \left(\frac{pt}{t} =\right) p$.

110. The

110. The Trapezias $FGhR$, $PDFR$, $frtC$, are ſimilar, and conſequently their correſponding parts are proportional.

That is, $\left\{\begin{matrix} \frac{FG}{PD} = \frac{Gh}{DF} = \frac{hR}{FR} = \frac{RF}{RP} \\ \overline{fr} = \overline{rt} = \overline{tC} = \overline{Cf} \end{matrix}\right\}$

For the triangles RFh, RPF, and FhG, FPD, are ſimilar.

111. The Trapezias $C\varrho PD$, $tCMP$ are ſimilar, and conſequently their correſponding parts are proportional.

That is, $\frac{C\varrho}{tC} = \frac{\varrho P}{CM} = \frac{PD}{PM} = \frac{DC}{Pt}$.

112. And CR, Cr, are parallel to Pf, PF, and equal to CA.

For $RTf = CTP = rTF$.

113. Let $\Sigma' =$ ſine of the $\angle tPf$, or TPF; $R =$ tabular radius.

Then $\frac{R}{\Sigma'} = \left(\frac{Pf}{fr} =\right) \frac{C}{c} = \frac{\sqrt{zv}}{c}$.

Put $\Sigma =$ ſine of the $\angle PCQ$, made by any diameter and its ordinate.

Then $\frac{TT}{CC} = \frac{1}{2}ff \pm \sqrt{\frac{1}{4}f^4 \backsim \frac{r^2t^2c^2}{\Sigma^2}}$; $\frac{tt}{cc} = \frac{1}{2}FF \pm \sqrt{\frac{1}{4}F^4 \backsim \frac{T^2C^2\Sigma^2}{rr}}$.

114. Let the parallels $PF\Pi$, $C\beta r$, $pf\pi$ be drawn, cutting the curve in Π, β, π; and ordinately applied to ſome diameter (2τ), whoſe parameter is 2π, and ſemiconjugate $C\beta = \chi$, to which $P\pi$ is ordinately applied at δ.

Then $C\delta = \left(\frac{PF + \pi f}{2} = \frac{PF + F\Pi}{2} =\right) \frac{1}{2}z + \frac{1}{2}z' = \frac{1}{2}P\Pi = \left(\frac{\overline{C\beta}^2}{Cr} =\right) \frac{\overline{C\beta}^2}{AC} = \frac{\chi\chi}{t} = \frac{\pi\tau}{t}$.

115. $\overline{C\beta}^2 = (\frac{1}{2}AC \times P\Pi =) \frac{1}{2}t \times \overline{z + z'} = \left(\overline{AC}^2 \times \frac{PF\Pi}{APa} =\right) \frac{tt}{cc}zz' = \frac{t}{p}zz'$.

116. $\frac{FP}{F\Pi} = \frac{z}{z'} = \frac{RD}{F\Delta} = \frac{tz - cc}{cc - tz'} = \frac{z - p}{p - z'}$.

117. $PF\Pi = \frac{pzz}{2z - p} = \frac{1}{2}p \times \overline{z + z'} = \frac{1}{2}p \times P\Pi$.

118. $P\Pi = z + z' = \frac{2zz}{2z - p} = \frac{2\pi\tau}{t} = \frac{\tau}{t} \times 2\pi$.

$$\overline{CB}^2 = cc = pt = \pm tt \mp ff = \pm \overline{t \mp f} \times \overline{t + f} = AFa = VAU = tCd = CFG$$
$$= PM \times C\varrho = FR \times fr = AN \times an = al \times AL = AC \times F\Phi = tt \times \frac{PM}{Pm}$$
$$= tt \times \frac{DM}{DC} = tt \times \frac{dC}{dm} = tt \times \frac{\overline{PM}^2}{\overline{CQ}^2} = tt \times \frac{\overline{PD}^2}{ADa} = tt \times \frac{\overline{PD}^2}{TDC} = ff \times \frac{PD}{Cm}$$
$$= \overline{\frac{CQ \times C\varrho}{AC}}\Big|^2 = \overline{\frac{AN \times CQ}{PN}}\Big|^2 = AMa - FMf = FPf - RPr = ff \times \frac{t}{t - p}$$
$$= TT + CC - tt = \frac{tt}{xx} \times \overline{zv + xx - tt} = \tfrac{1}{2} zv \pm \sqrt{\tfrac{1}{4} z^2 v^2 ffyy}.$$

$$\overline{CQ}^2 = CC = P \times T = MPm = NPn = TPt = FPf = CP\eta = Lpl$$
$$= PN \times pL = CD \times TM = PD \times tm = \frac{tt}{cc} \times \overline{PM}^2 = \frac{cc}{tt} \times \overline{Pm}^2 = \frac{cc}{ff} \times \overline{fm}^2$$
$$= \frac{tt}{ff} \times FMf = \overline{\frac{ct}{ff} \times Mm}\Big|^2 = \overline{\frac{ACB}{C\varrho}}\Big|^2 = \overline{PM}^2 + FMf = \overline{BC}^2 + RPr$$
$$= tt + cc - TT = tt - \frac{ff}{tt} xx = \frac{tt}{cc} yy + \frac{cc}{tt} xx = \frac{DC}{DT} \times \overline{PT}^2.$$

$$\overline{CF}^2 = ff = tt - cc = tt - tp = \frac{cc}{p} \times \overline{t - p} = \frac{tt}{xx} \times mm = P\varrho \times Tt$$
$$= \frac{tt}{CC} \times FMf = \frac{cc}{CC} \times \overline{fm}^2 = \frac{mm}{CC} \times FTf = \frac{mm}{CC} \times NTn.$$

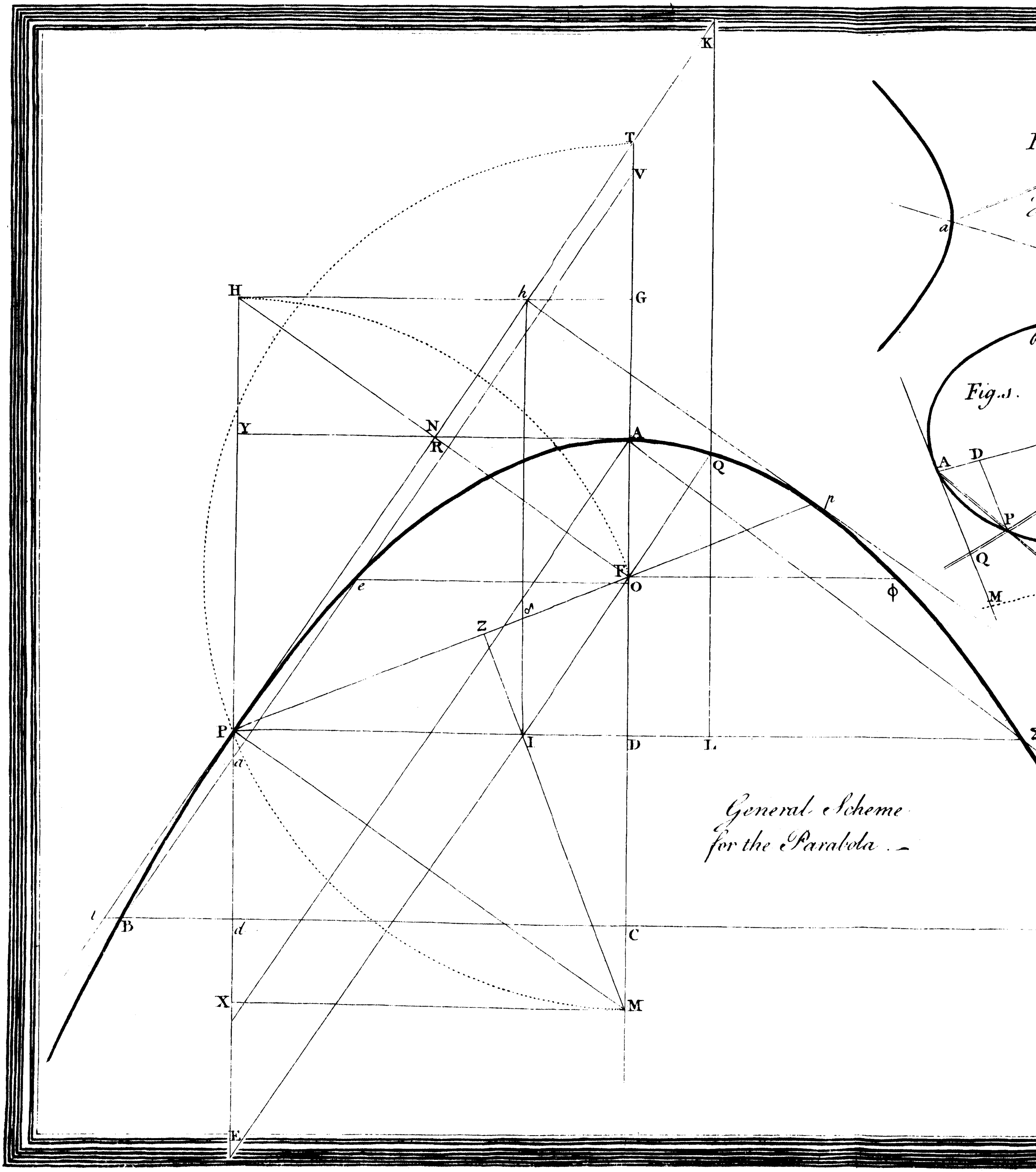
The semi-parameter (p) to the greater axis (Aa) is equal to

$$F\Phi = PZ = \frac{AFa}{AC} = \frac{VaU}{AC} = \frac{CFG}{AC} = \frac{tCd}{AC} = \frac{PF\Pi}{\frac{1}{2}P\Pi} = \frac{PM \times C\varrho}{AC}$$
$$= \frac{FR \times fr}{AC} = \frac{AN \times an}{AC} = \frac{AC \times PM}{Pm} = \frac{AC \times DM}{DC} = \frac{AC \times dC}{dm} = \frac{BC \times PM}{CQ}$$
$$= \frac{BC \times CQ}{Pm} = \frac{\overline{BC}^2}{AC} = \frac{cc}{t} = \frac{ttyy}{sx} = \frac{\pm tt \mp ff}{t} = \frac{cc - yy}{xx} \times t$$
$$= \frac{CC + TT - tt}{t} = \frac{CC + xx - tt}{xx} \times t = \frac{2t - z}{tz} \times \overline{FR}^2 = \frac{2t - v}{tv} \times \overline{fr}^2.$$

The semi-parameter (P) to any diameter (Pp) is equal to

$$P\eta = \frac{\overline{CQ}^2}{PC} = \frac{CC}{T} = \frac{MPm}{PC} = \frac{NPn}{PC} = \frac{TPt}{PC} = \frac{FPf}{PC} = \frac{Lpl}{PC} = \frac{PN \times pL}{PC}$$
$$= \frac{CD \times TM}{PC} = \frac{PD \times tm}{PC} = \frac{PT \times PN}{PV} = \frac{tt}{cc} \times \frac{\overline{PM}^2}{PC} = \frac{cc}{tt} \times \frac{Pm^2}{PC}$$
$$= \frac{cc}{ff} \times \frac{\overline{fm}^2}{PC} = \frac{tt}{ff} \times \frac{PMf}{PC} = \frac{tt + cc - TT}{T}.$$ [See TAB. XIV, XV.]

XXXVII. *An*



General Scheme
for the Parabola.

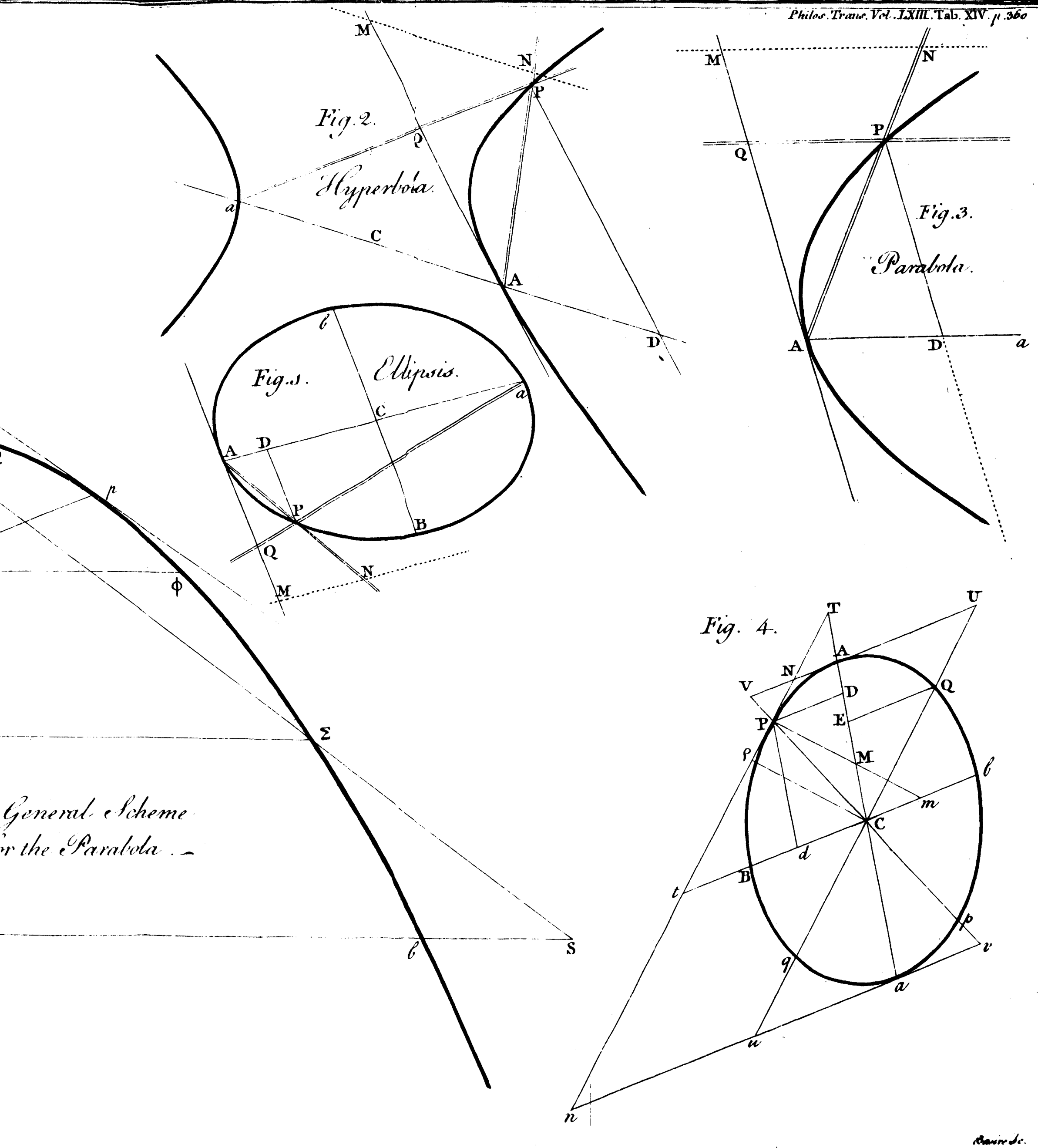


Fig. 2.
Hyperbola.
Fig. 3.
Parabola.
Fig. 1.
Ellipsis.
Fig. 4.
General Scheme
for the Parabola.
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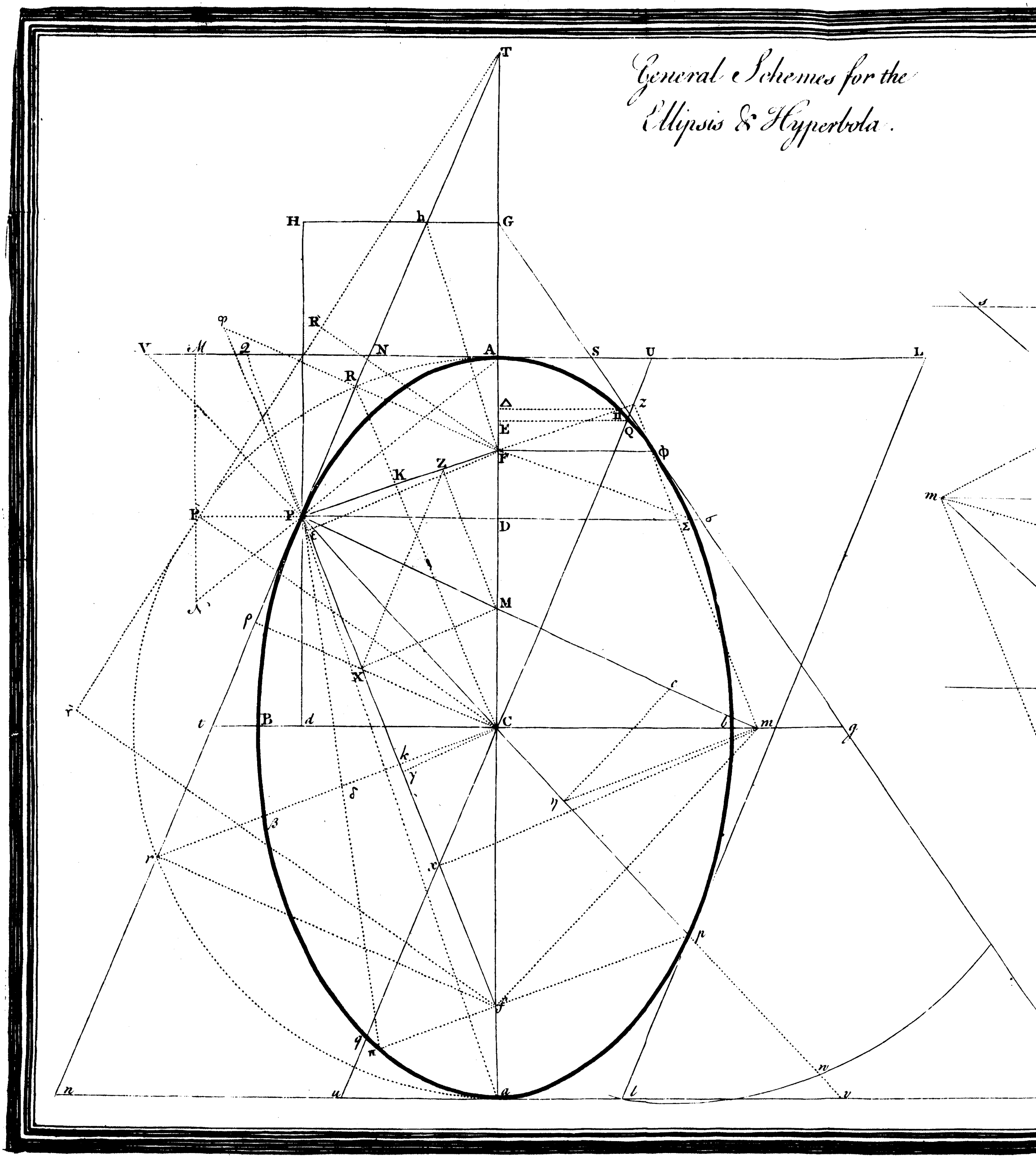
General Schemes for the Ellipsis & Hyperbola.



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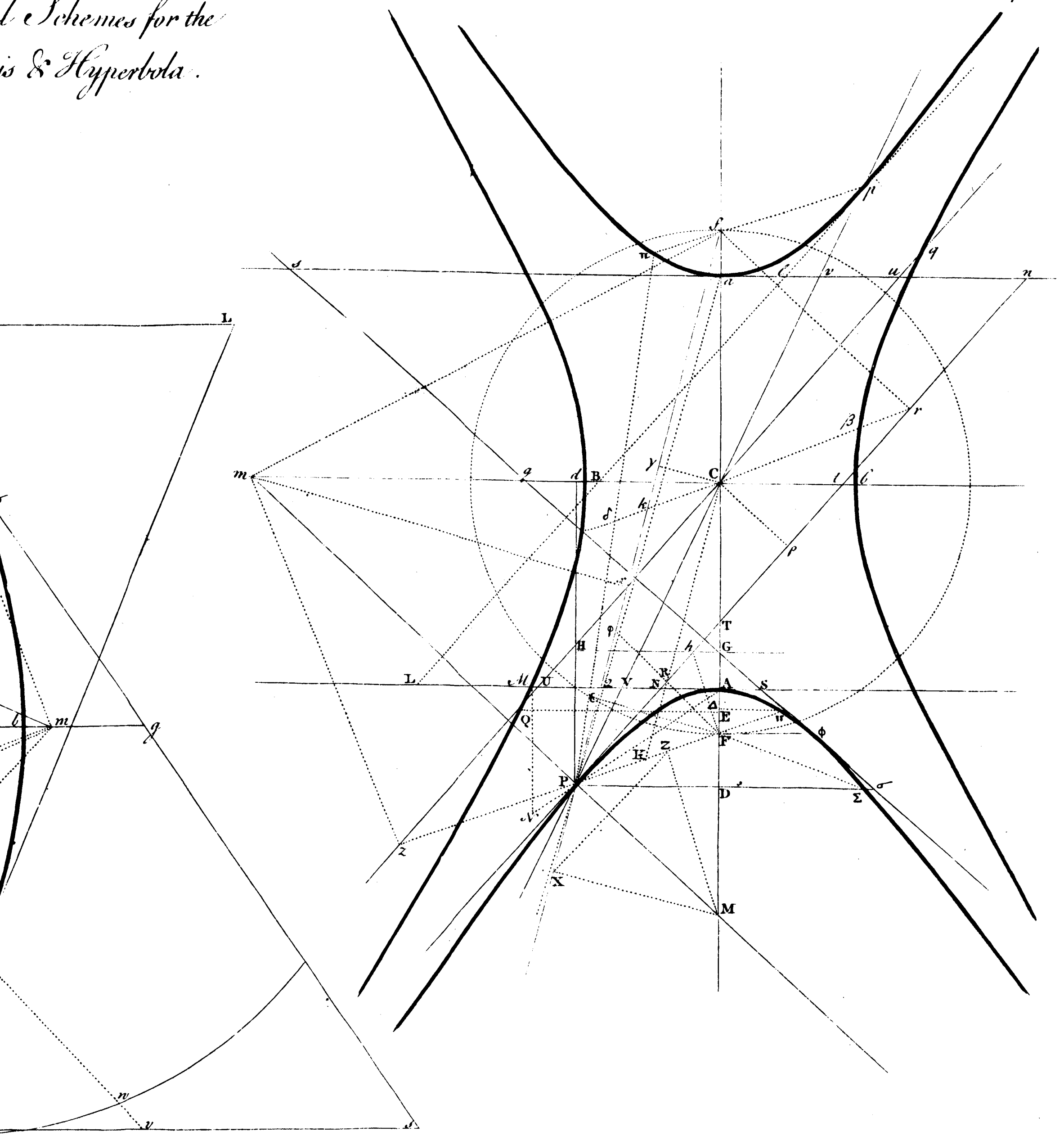
is & Hyperbola.



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